PVP 14

Code: EC3T2

II B.Tech - I Semester – Regular Examinations – December 2015

PROBABILITY THEORY AND STOCHASTIC PROCESS
(ELECTRONICS AND COMMUNICATION ENGINEERING)

Duration: 3 hours Max. Marks: 70

PART – A

Answer ***all*** the questions. All questions carry equal marks

11x 2 = 22 M

1. a) Find the total number of non-empty events of a sample space, when a fair die is rolled.

b) What is the probability of getting a red colored king card, when a card is drawn at random from a well-shuffled deck of 52 playing cards.

c) If X is a random variable then find

i) $P\{X=-\infty\}$ ii) $P\{X=\infty\}$.

d) Find the mean of a random variable X if $f_X(x)=u(x)e^{-x}$ where $u(.)$ is a unit step function.

e) Write any four properties of Joint Distribution function.

f) Find the value of k so that

$$f(x,y)=\begin{cases} k\,e^{-x}\text{siny}, & 0\le x<\infty \text{ and } 0\le y\le\frac{\pi}{2}. \\ 0 & \text{, elesewhere} \end{cases}$$

is a valid joint density function.

g) Find $E[XY]$, if X,Y are statistically independent random variables with $\overline{X}=2$ and $\overline{Y}=3$.

h) Define stationary random process.

i) Write any four properties of Power Density Spectrum.

j) Define Band-Limited Processes

k) Define Average Noise Figure.

PART – B

Answer any ***THREE*** questions. All questions carry equal marks. 3 x 16 = 48 M

2. a) Define Total probability and state Baye's theorem. 4 M

b) Explain when we say that three events are statistically independent? 4 M

c) A missile can be launched if two relays A and B both have failed. The probabilities of A and B failing are known to be 0.02 and 0.04, respectively. It is also known that B is more likely to fail (probability 0.07) if A has failed.

i) What is the probability of an accidental missile launch?

ii) What is the probability that A will fail if B has failed?

iii) Are the events "A fails" and "B fails" statistically independent? 8 M

3. a) Write any four properties of Density function $f_X(x)$. 4 M

b) Assume that the height of trees above the ground at some location is a gaussian random variable X with mean, $\mu = 5$ ft. and standard deviation , $\sigma = 1$ ft. Find the probability that the trees will be higher than 6 ft. 4 M

c) Find the value of k, so that $f_X(x) = \frac{k}{3^x}$, for $x = 0,1,2,3,...$ is a valid density function, and hence find mean, variance and standard deviation of the discrete random variable X.

8 M

4. a) Find a constant b (in terms of a) so that the function

$$f_{X,Y}(x,y) = \begin{cases} b e^{(x+y)}, & 0 < x < a \text{ and } 0 < y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

is a valid joint density function and hence find an expression for the joint distribution function. 8 M

b) Statistically independent random variables X and Y have respective densities $f_X(x) = 5u(x)e^{-5x}$, $f_Y(y) = 2u(y)e^{-2y}$. Find the density of the sum W=X+Y. 8 M

5. a) If X(t) is a stationary random process having a mean value $E[X(t)] = 3$ and autocorrelation function $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$, find i) the mean value and

ii) the variance of the random variable $Y = \int_0^2 X(t)\, dt$ 8 M

b) A random process is given by $X(t) = A_0 \cos(\omega_0 t + \Theta)$, where A_0, ω_0 are real constants, and Θ is a random variable uniformly distributed on the interval $\left(0, \frac{\pi}{2}\right)$. Find the average power P_{XX}. 8 M

6. a) A random process $X(t) = A \sin(\omega_0 t + \Theta)$ where A and ω_0 are real positive constants and Θ is a random variable uniformly distributed on the interval $(-\pi, \pi)$, is applied to the network having an impulse response $h(t) = W u(t) e^{-Wt}$, where $W > 0$ and $u(.)$ is a unit step function. Find an expression for the network's response process. 8 M

b) Explain Resistive (Thermal) Noise Source, Arbitrary Noise Sources, Effective Noise Temperature. 8 M